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THE DRINFELD CENTER OF THE CATEGORY OF MACKEY FUNCTORS

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We determine the center of the tensor category of Mackey functors for a finite group. Details are in [5].

1. The center of a tensor category

The center of a tensor category was defined by Drinfeld, Joyal and Street ([3]), and Magid ([4]). We review the definition. Let \mathcal{A} be a tensor category over a field. The tensor product of objects $A, B \in \mathcal{A}$ is denoted by $A \otimes B$, and the unit object of \mathcal{A} is denoted by I .

The center $\mathbb{Z}(\mathcal{A})$ is a category defined as follows. An object of $\mathbb{Z}(\mathcal{A})$ is a pair (A, θ) , where $A \in \mathcal{A}$ and θ is a family of isomorphisms $\theta_B: B \otimes A \rightarrow A \otimes B$ for all $B \in \mathcal{A}$ satisfying the conditions

$$\begin{aligned}\theta_{B \otimes B'} &= (\theta_B \otimes 1) \circ (1 \otimes \theta_{B'}) \quad \text{for all } B, B' \in \mathcal{A}, \\ \theta_I &= 1.\end{aligned}$$

A morphism $(A, \theta) \rightarrow (A', \theta')$ of $\mathbb{Z}(\mathcal{A})$ is a morphism $f: A \rightarrow A'$ of \mathcal{A} satisfying

$$(f \otimes 1) \circ \theta_B = \theta'_B \circ (1 \otimes f) \quad \text{for all } B \in \mathcal{A}.$$

2. Mackey functors

We review the definition of a Mackey functor ([1], [2]). Let G be a finite group. Denote by \mathcal{S} the category of finite G -sets. For $X, Y \in \mathcal{S}$, we write the direct product $X \times Y$ as XY , and the disjoint sum of X and Y as $X + Y$. Let k be a field. Denote by \mathcal{V} the category of vector spaces over k .

A Mackey functor M on \mathcal{S} consists of k -vector spaces $M(X)$ for all G -sets X and linear maps $f_*: M(X) \rightarrow M(Y)$ and $f^*: M(Y) \rightarrow M(X)$ for all G -maps $f: X \rightarrow Y$ satisfying the following conditions:

- (i) $M(X)$ and f_* form a functor $\mathcal{S} \rightarrow \mathcal{V}$.
- (ii) $M(X)$ and f^* form a functor $\mathcal{S}^{\text{op}} \rightarrow \mathcal{V}$.

(iii) For a pullback diagram

$$\begin{array}{ccc} X & \xrightarrow{p} & X' \\ f \downarrow & & \downarrow f' \\ Y & \xrightarrow{q} & Y' \end{array}$$

in \mathcal{S} , the diagram

$$\begin{array}{ccc} M(X) & \xleftarrow{p^*} & M(X') \\ f_* \downarrow & & \downarrow f'_* \\ M(Y) & \xleftarrow{q^*} & M(Y') \end{array}$$

is commutative.

(iv) Let $i_1: X_1 \rightarrow X_1 + X_2$ and $i_2: X_2 \rightarrow X_1 + X_2$ be the inclusion maps in \mathcal{S} . Then the maps

$$\begin{aligned} (i_{1*}, i_{2*}): M(X_1) \oplus M(X_2) &\rightarrow M(X_1 + X_2), \\ (i_1^*, i_2^*): M(X_1 + X_2) &\rightarrow M(X_1) \oplus M(X_2) \end{aligned}$$

are inverse to each other.

(v) $M(\emptyset) = 0$.

The category of Mackey functors on \mathcal{S} is denoted by $\mathbf{M}(\mathcal{S})$.

We use the following fact later. If M is a Mackey functor and $i: Y \rightarrow X$ is a monomorphism in \mathcal{S} , then the composite

$$M(Y) \xrightarrow{i_*} M(X) \xrightarrow{i^*} M(Y)$$

is the identity. So the composite

$$M(X) \xrightarrow{i^*} M(Y) \xrightarrow{i_*} M(X)$$

is an idempotent endomorphism.

The category $\mathbf{M}(\mathcal{S})$ is a tensor category. Its tensor product is defined as follows. Let $M, M', M'' \in \mathbf{M}(\mathcal{S})$. A bilinear morphism $\phi: (M, M') \rightarrow M''$ is a family of linear maps $\phi_{X,Y}: M(X) \otimes M'(Y) \rightarrow M''(XY)$ which commute with f_* and f^* for the both variables X, Y . Given $M, M' \in \mathbf{M}(\mathcal{S})$, there exists a bilinear morphism $(M, M') \rightarrow M_0$ which is universal among all bilinear morphisms $(M, M') \rightarrow M''$. We define $M_0 = M \otimes M'$.

If \mathcal{C} is a category with pullbacks and sums, Mackey functors on \mathcal{C} are similarly defined. We denote by $\mathbf{M}(\mathcal{C})$ the category of Mackey functors on \mathcal{C} .

3. The main result

We define a category \mathcal{T}_{c*} . An object of \mathcal{T}_{c*} is a pair (X, a) of $X \in \mathcal{S}$ and an automorphism $a: X \rightarrow X$ of \mathcal{S} such that a leaves all G -orbits in X stable. A morphism $(X, a) \rightarrow (X', a')$ of \mathcal{T}_{c*} is a morphism $f: X \rightarrow X'$ of \mathcal{S} such that $f \circ a = a' \circ f$.

The category \mathcal{T}_{c*} has pullbacks and sums, so the category $\mathbf{M}(\mathcal{T}_{c*})$ is defined.

A construction of pullback in \mathcal{T}_{c*} is as follows. Given a diagram

$$\begin{array}{ccc} & (Y, b) & \\ & \downarrow & \\ (X, a) & \longrightarrow & (Z, c) \end{array}$$

in \mathcal{T}_{c*} , form a pullback

$$\begin{array}{ccc} W & \longrightarrow & Y \\ \downarrow & & \downarrow \\ X & \longrightarrow & Z \end{array}$$

in \mathcal{S} . The maps $a: X \rightarrow X$ and $b: Y \rightarrow Y$ induce $d: W \rightarrow W$. Put

$$V = \bigcup \{U \mid U \text{ is a } G\text{-orbit in } W, d(U) = U\}$$

and $e = d|_V$. Then

$$\begin{array}{ccc} (V, e) & \longrightarrow & (Y, b) \\ \downarrow & & \downarrow \\ (X, a) & \longrightarrow & (Z, c) \end{array}$$

is a pullback in \mathcal{T}_{c*} .

Our result is

Theorem. *An equivalence of categories $\mathbf{Z}(\mathbf{M}(\mathcal{S})) \simeq \mathbf{M}(\mathcal{T}_{c*})$.*

By definition an object of $\mathbf{Z}(\mathbf{M}(\mathcal{S}))$ is a pair (M, θ) of $M \in \mathbf{M}(\mathcal{S})$ and a family θ of isomorphisms $\theta_{M'}: M' \otimes M \rightarrow M \otimes M'$ for all $M' \in \mathbf{M}(\mathcal{S})$ satisfying certain conditions. We may also regard an object of $\mathbf{Z}(\mathbf{M}(\mathcal{S}))$ as a pair (M, ω) of $M \in \mathbf{M}(\mathcal{S})$ and a family ω of isomorphisms $\omega_{X,Y}: M(XY) \rightarrow M(YX)$ for all $X, Y \in \mathcal{S}$ satisfying (i)–(iii):

- (i) $\omega_{X,Y}$ is natural in X, Y .
- (ii) The diagram

$$\begin{array}{ccc} M(XYZ) & \xrightarrow{\omega_{X,YZ}} & M(YZX) \\ \omega_{XY,Z} \searrow & & \downarrow \omega_{Y,ZX} \\ & & N(ZXY) \end{array}$$

commutes for all $X, Y, Z \in \mathcal{S}$.

(iii) $\omega_{1,X} = 1$ for a one-element G -set 1 .

The equivalence of the theorem is given as follows. Let $(M, \omega) \in \mathbb{Z}(\mathbb{M}(\mathcal{S}))$. For an object $(X, a) \in \mathcal{T}_{c*}$, define $L(X, a)$ as the pullback

$$\begin{array}{ccc} L(X, a) & \rightarrow & M(X) \\ & & \downarrow (a,1)_* \\ & & M(XX) \\ \downarrow & & \downarrow \omega_{X,X} \\ M(X) & \xrightarrow{(1,1)_*} & M(XX) \end{array}$$

where $(a, 1): X \rightarrow XX$ is the map $x \mapsto (a(x), x)$, and $(1, 1): X \rightarrow XX$ is the diagonal map. Then the assignment $(X, a) \mapsto L(X, a)$ becomes a Mackey functor L on \mathcal{T}_{c*} . The functor $(M, \omega) \mapsto L$ gives the equivalence $\mathbb{Z}(\mathbb{M}(\mathcal{S})) \simeq \mathbb{M}(\mathcal{T}_{c*})$.

4. Outline of the proof

The equivalence of the theorem is obtained as the composite of equivalences

$$\mathbb{Z}(\mathbb{M}(\mathcal{S})) \underset{(1)}{\simeq} {}_S\mathbb{M}(\mathcal{S}, \mathcal{S})_S \underset{(2)}{\simeq} \mathbb{M}_0(\mathcal{W}') \underset{(3)}{\simeq} \mathbb{M}(\mathcal{W}_{ic*}) \underset{(4)}{\simeq} \mathbb{M}(\mathcal{T}_{c*}).$$

We will sketch each equivalence in order.

(1) $\mathbb{Z}(\mathbb{M}(\mathcal{S})) \simeq {}_S\mathbb{M}(\mathcal{S}, \mathcal{S})_S$.

A *bi-Mackey functor* N on \mathcal{S} consists of vector spaces $N(X, Y)$ for all G -sets X and Y , and linear maps

$$\begin{aligned} \langle f, g \rangle_* &: N(X, Y) \rightarrow N(X', Y'), \\ \langle f, g \rangle^* &: N(X', Y') \rightarrow N(X, Y) \end{aligned}$$

for all G -maps $f: X \rightarrow X'$ and $g: Y \rightarrow Y'$ satisfying (i)–(ix):

- (i) The collection of $N(X, Y)$ and $\langle f, g \rangle_*$ forms a functor $\mathcal{S} \times \mathcal{S} \rightarrow \mathcal{V}$.
- (ii) The collection of $N(X, Y)$ and $\langle f, g \rangle^*$ forms a functor $\mathcal{S}^{\text{op}} \times \mathcal{S}^{\text{op}} \rightarrow \mathcal{V}$.
- (iii) For G -maps $f: X \rightarrow X'$ and $g: Y \rightarrow Y'$, the diagrams

$$\begin{array}{ccccc} N(X, Y) & \xrightarrow{\langle f, 1 \rangle_*} & N(X', Y) & & N(X, Y) & \xleftarrow{\langle f, 1 \rangle^*} & N(X', Y) \\ \langle 1, g \rangle^* \uparrow & & \uparrow \langle 1, g \rangle^* & & \langle 1, g \rangle_* \downarrow & & \downarrow \langle 1, g \rangle_* \\ N(X, Y') & \xrightarrow{\langle f, 1 \rangle_*} & N(X', Y') & & N(X, Y') & \xleftarrow{\langle f, 1 \rangle^*} & N(X', Y') \end{array}$$

are commutative.

(iv) If

$$\begin{array}{ccc} X_1 & \xrightarrow{f_1} & X'_1 \\ p \downarrow & & \downarrow p' \\ X_2 & \xrightarrow{f_2} & X'_2 \end{array}$$

is a pullback diagram, then

$$\begin{array}{ccc} N(X_1, Y) & \xrightarrow{\langle f_1, 1 \rangle_*} & N(X'_1, Y) \\ \langle p, 1 \rangle^* \uparrow & & \uparrow \langle p', 1 \rangle^* \\ N(X_2, Y) & \xrightarrow{\langle f_2, 1 \rangle_*} & N(X'_2, Y) \end{array}$$

is commutative.

(v) The analogue of (iv) for the second variable.

(vi) Let $i_1: X_1 \rightarrow X_1 + X_2$, $i_2: X_2 \rightarrow X_1 + X_2$ denote the inclusion maps. Then the maps

$$\begin{aligned} (\langle i_1, 1 \rangle_*, \langle i_2, 1 \rangle_*) &: N(X_1, Y) \oplus N(X_2, Y) \rightarrow N(X_1 + X_2, Y), \\ (\langle i_1, 1 \rangle^*, \langle i_2, 1 \rangle^*) &: N(X_1 + X_2, Y) \rightarrow N(X_1, Y) \oplus N(X_2, Y) \end{aligned}$$

are inverse to each other.

(vii) The analogue of (vi) for the second variable.

(viii) $N(\emptyset, Y) = 0$.

(ix) $N(X, \emptyset) = 0$.

A *bi-Mackey functor on \mathcal{S} with two-sided action* is a bi-Mackey functor N on \mathcal{S} equipped with maps

$$\begin{aligned} Z! &: N(X, Y) \rightarrow N(ZX, ZY), \\ !Z &: N(X, Y) \rightarrow N(XZ, YZ) \end{aligned}$$

for $X, Y, Z \in \mathcal{S}$ satisfying (i)–(ix):

(i) For G -maps $f: X \rightarrow X'$ and $g: Y \rightarrow Y'$, the diagrams

$$\begin{array}{ccc} N(X, Y) & \xrightarrow{\langle f, g \rangle_*} & N(X', Y') \\ !Z \downarrow & & \downarrow !Z \\ N(XZ, YZ) & \xrightarrow{\langle 1f, 1g \rangle_*} & N(X'Z, Y'Z) \end{array}$$

and

$$\begin{array}{ccc} N(X, Y) & \xleftarrow{\langle f, g \rangle^*} & N(X', Y') \\ \downarrow !Z & & \downarrow !Z \\ N(XZ, YZ) & \xleftarrow{\langle 1f, 1g \rangle^*} & N(X'Z, Y'Z) \end{array}$$

are commutative.

(ii) For G -map $h: Z \rightarrow Z'$, the diagrams

$$\begin{array}{ccc} N(X, Y) & \xrightarrow{!Z} & N(XZ, YZ) \\ \downarrow !Z' & & \downarrow \langle 1, 1h \rangle^* \\ N(XZ', YZ') & \xrightarrow{\langle 1h, 1 \rangle^*} & N(XZ, YZ') \end{array}$$

and

$$\begin{array}{ccc} N(X, Y) & \xrightarrow{!Z} & N(XZ, YZ) \\ \downarrow !Z' & & \downarrow \langle 1h, 1 \rangle^* \\ N(XZ', YZ') & \xrightarrow{\langle 1, 1h \rangle^*} & N(XZ', YZ) \end{array}$$

are commutative.

(iii) The diagram

$$\begin{array}{ccc} N(X, Y) & \xrightarrow{!Z} & N(XZ, YZ) \\ !ZZ' \searrow & & \downarrow !Z' \\ & & N(XZZ', YZZ') \end{array}$$

is commutative.

(iv) For a one-element G -set 1 ,

$$!1: N(X, Y) \rightarrow N(X1, Y1)$$

is the identity.

(v)–(viii) The analogue of (i)–(iv) for $Z!$.

(ix) The diagram

$$\begin{array}{ccc} N(X, Y) & \xrightarrow{Z!} & N(ZX, ZY) \\ !W \downarrow & & \downarrow !W \\ N(XW, YW) & \xrightarrow{Z!} & N(ZXW, ZYW) \end{array}$$

is commutative.

The category of bi-Mackey functors on \mathcal{S} with two-sided action is denoted by ${}_S\mathbf{M}(\mathcal{S}, \mathcal{S})_S$.

Proposition. *We have an equivalence $\mathbb{Z}(\mathbb{M}(S)) \simeq {}_S\mathbb{M}(S, S)_S$.*

This equivalence takes an object $(M, \omega) \in \mathbb{Z}(\mathbb{M}(S))$ to an object $N \in {}_S\mathbb{M}(S, S)_S$ defined as follows. For $X, Y \in S$

$$N(X, Y) = M(XY).$$

The operation

$$!Z: N(X, Y) \rightarrow N(XZ, YZ)$$

is the composite

$$M(XY) \xrightarrow{(11p)^*} M(XYZ) \xrightarrow{(11\Delta)^*} M(XYZZ) \xrightarrow{(1\tau 1)^*} M(XZYZ),$$

where $\Delta: Z \rightarrow ZZ$ is the diagonal map and $\tau: YZ \rightarrow ZY$ is the transposition.

The operation

$$Z!: N(X, Y) \rightarrow N(ZX, ZY)$$

is the composite

$$M(XY) \xrightarrow{(p11)^*} M(ZXY) \xrightarrow{(\Delta 11)^*} M(ZZXY) \xrightarrow{\omega_{Z, ZXY}} M(ZXYZ) \xrightarrow{(11\tau)^*} M(ZXZY).$$

$$(2) {}_S\mathbb{M}(S, S)_S \simeq \mathbb{M}_0(\mathcal{W}').$$

Let \mathcal{W}' be the category whose objects are diagrams

$$\begin{array}{ccc} & U & \\ X & \swarrow \searrow & Y \\ & V & \end{array}$$

of G -sets such that the induced maps $U \rightarrow XY$, $V \rightarrow XY$ are injective, and morphisms are natural ones. This has pullbacks and sums, so one has the category $\mathbb{M}(\mathcal{W}')$ of Mackey functors on \mathcal{W}' .

Suppose that $(M, \omega) \in \mathbb{Z}(\mathbb{M}(S))$ corresponds to $N \in {}_S\mathbb{M}(S, S)_S$ under the equivalence (1). Let

$$\mathbf{X} = \left(\begin{array}{ccc} & U & \\ X & \swarrow \searrow & Y \\ & V & \end{array} \right) \in \mathcal{W}'.$$

As noted after the definition of a Mackey functor, the injection $V \rightarrow XY$ determines an idempotent endomorphism on $M(XY)$. As $M(XY) = N(X, Y)$, this is an idempotent endomorphism on $N(X, Y)$, which we denote by

$$e^R(X \leftarrow V \rightarrow Y).$$

Similarly the injection $U \rightarrow YX$ determines an idempotent endomorphism on $M(YX)$. Through the isomorphism $\omega_{X, Y}: M(XY) \rightarrow M(YX)$ and $M(XY) = N(X, Y)$, this yields an idempotent endomorphism on $N(X, Y)$, which we denote by

$$e^L(X \leftarrow U \rightarrow Y).$$

Lemma. *The idempotent endomorphisms $e^L(X \leftarrow U \rightarrow Y)$ and $e^R(X \leftarrow V \rightarrow Y)$ on $N(X, Y)$ commute with each other.*

We set

$$H(\mathbf{X}) = \text{Im } e^L(X \leftarrow U \rightarrow Y) \cap \text{Im } e^R(X \leftarrow V \rightarrow Y)$$

Then the assignment $\mathbf{X} \mapsto H(\mathbf{X})$ becomes a Mackey functor H on \mathcal{W}' . We thus obtain a functor

$$\begin{aligned} {}_S\mathbf{M}(\mathcal{S}, \mathcal{S})_S &\rightarrow \mathbf{M}(\mathcal{W}') \\ N &\mapsto H. \end{aligned}$$

This is fully faithful. To describe its image, we define a full subcategory $\mathbf{M}_0(\mathcal{W}')$ of $\mathbf{M}(\mathcal{W}')$.

An object of $\mathbf{M}_0(\mathcal{W}')$ is an object H of $\mathbf{M}(\mathcal{W}')$ which satisfies (i)–(viii):

(i) Suppose that

$$\mathbf{X} = \left(\begin{array}{ccc} & U_1 + U_2 & \\ X & \swarrow \quad \searrow & Y \\ & V & \end{array} \right)$$

is an object of \mathcal{W}' . Put

$$\mathbf{X}_1 = \left(\begin{array}{ccc} & U_1 & \\ X & \swarrow \quad \searrow & Y \\ & V & \end{array} \right), \quad \mathbf{X}_2 = \left(\begin{array}{ccc} & U_2 & \\ X & \swarrow \quad \searrow & Y \\ & V & \end{array} \right)$$

and let $i_1: \mathbf{X}_1 \rightarrow \mathbf{X}$, $i_2: \mathbf{X}_2 \rightarrow \mathbf{X}$ be the natural injections. Then the maps

$$\begin{aligned} (i_{1*}, i_{2*}): H(\mathbf{X}_1) \oplus H(\mathbf{X}_2) &\rightarrow H(\mathbf{X}), \\ (i_1^*, i_2^*): H(\mathbf{X}) &\rightarrow H(\mathbf{X}_1) \oplus H(\mathbf{X}_2) \end{aligned}$$

are inverse to each other.

(ii)

$$H \left(\begin{array}{ccc} & \emptyset & \\ X & \swarrow \quad \searrow & Y \\ & V & \end{array} \right) = 0.$$

(iii) The analogue of (i) for the V -component.

(iv) The analogue of (ii) for the V -component.

(v) Let

$$\mathbf{X}_1 = \left(\begin{array}{ccc} & U_1 & \\ X_1 & \swarrow \quad \searrow & Y \\ & V_1 & \end{array} \right), \quad \mathbf{X}_2 = \left(\begin{array}{ccc} & U_2 & \\ X_2 & \swarrow \quad \searrow & Y \\ & V_2 & \end{array} \right)$$

be objects of \mathcal{W}' . Put

$$\mathbf{X} = \left(\begin{array}{ccc} & U_1 + U_2 & \\ X_1 + X_2 & \swarrow \quad \searrow & Y \\ & V_1 + V_2 & \end{array} \right)$$

and let $j_1: \mathbf{X}_1 \rightarrow \mathbf{X}$, $j_2: \mathbf{X}_2 \rightarrow \mathbf{X}$ be the natural injections. Then the maps

$$\begin{aligned} (j_{1*}, j_{2*}): H(\mathbf{X}_1) \oplus H(\mathbf{X}_2) &\rightarrow H(\mathbf{X}), \\ (j_1^*, j_2^*): H(\mathbf{X}) &\rightarrow H(\mathbf{X}_1) \oplus H(\mathbf{X}_2) \end{aligned}$$

are inverse to each other.

(vi) The analogue of (v) for the Y -component.

(vii) Let

$$\mathbf{X} = \left(\begin{array}{ccc} & U & \\ X & \xleftarrow{a} \quad \xrightarrow{b} & Y \\ & V & \end{array} \right)$$

be an object of \mathcal{W}' . Let

$$\begin{array}{ccc} V_1 & \xrightarrow{(c_1, d_1)} & UU \\ e \downarrow & & \downarrow ab \\ V & \xrightarrow{(c, d)} & XY \end{array}$$

be a pullback. Put

$$\mathbf{U} = \left(\begin{array}{ccc} & U & \\ U & \xleftarrow{1} \quad \xrightarrow{1} & U \\ & V_1 & \end{array} \right)$$

and

$$\mathbf{a} = \begin{pmatrix} 1 & \\ a & b \\ & e \end{pmatrix} : \mathbf{U} \rightarrow \mathbf{X}.$$

Then the maps

$$\begin{aligned} \mathbf{a}_*: H(\mathbf{U}) &\rightarrow H(\mathbf{X}), \\ \mathbf{a}^*: H(\mathbf{X}) &\rightarrow H(\mathbf{U}) \end{aligned}$$

are inverse to each other.

(viii) The analogue of (vii) for the V -component.

The functor ${}_S\mathbf{M}(\mathcal{S}, \mathcal{S})_{\mathcal{S}} \rightarrow \mathbf{M}(\mathcal{W}')$ constructed before has the image $\mathbf{M}_0(\mathcal{W}')$, and yields

Proposition. *An equivalence ${}_S\mathbb{M}(\mathcal{S}, \mathcal{S})_S \simeq \mathbb{M}_0(\mathcal{W}')$.*

(3) $\mathbb{M}_0(\mathcal{W}') \simeq \mathbb{M}(\mathcal{W}_{ic*})$.

Let \mathcal{W}_{ic*} be the full subcategory of \mathcal{W}' consisting of finite sums of diagrams

$$\begin{array}{ccc} & U & \\ X & \swarrow \searrow & Y \\ & V & \end{array}$$

in which X, Y, U, V are transitive G -sets and the four arrows are isomorphisms.

Lemma. *The inclusion functor $\mathcal{W}_{ic*} \rightarrow \mathcal{W}'$ has a right adjoint.*

Denote the inclusion $\mathcal{W}_{ic*} \rightarrow \mathcal{W}'$ by i and a right adjoint by R .

Proposition. *We have an equivalence $\mathbb{M}_0(\mathcal{W}') \simeq \mathbb{M}(\mathcal{W}_{ic*})$.*

Under the equivalence objects $H \in \mathbb{M}_0(\mathcal{W}')$ and $K \in \mathbb{M}(\mathcal{W}_{ic*})$ correspond if

$$H \cong K \circ R, \quad K \cong H \circ i.$$

(4) $\mathbb{M}(\mathcal{W}_{ic*}) \simeq \mathbb{M}(\mathcal{T}_{c*})$.

An object of the category \mathcal{T}_{c*} is a pair (X, a) of $X \in \mathcal{S}$ and an automorphism $a: X \rightarrow X$ such that a leaves all G -orbits stable. The functor

$$(X, a) \mapsto \left(\begin{array}{ccc} & X & \\ X & \swarrow \searrow & X \\ & X & \end{array} \right)$$

gives an equivalence $\mathcal{T}_{c*} \simeq \mathcal{W}_{ic*}$. This yields

Proposition. *An equivalence $\mathbb{M}(\mathcal{W}_{ic*}) \simeq \mathbb{M}(\mathcal{T}_{c*})$.*

Combining (1)–(4), we obtain $\mathbb{Z}(\mathbb{M}(\mathcal{S})) \simeq \mathbb{M}(\mathcal{T}_{c*})$.

References

1. S. Bouc, Green Functors and G -sets, Lecture Notes in Math. 1671, Springer-Verlag, Berlin, 1997.
2. A. Dress, Contribution to the theory of induced representations, in: Algebraic K-theory II, Lecture Notes in Math. 342, Springer-Verlag, 1973, pp.183–240.
3. A. Joyal and R. Street, Tortile Yang-Baxter operators in tensor categories, Journal of Pure and Applied Algebra 71 (1991), 43–51.
4. S. Majid, Representations, duals and quantum doubles of monoidal categories, Rend. Circ. Math. Palermo (2) Suppl. 26 (1991), 197–206.
5. D. Tambara, The Drinfeld center of the category of Mackey functors, in submission.